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A multilayer contact process

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Abstract. We introduce a contact model with evaporation and deposition of particles at rates p and $(1-p)$, respectively, per occupied lattice site; while the deposition probability on empty sites depends on the number of occupied nearest-neighbour sites. At large times t this model has three different phases, separated by two critical points ($p_{1c} = \frac{1}{2}$ and $p_{2c} = 0.6473 \pm 0.0003$). Such phases are: (i) The growth phase ($0 \leqslant p \leqslant \frac{1}{2}$). Here the mean value of particles per lattice site n and its fluctuations w always increase as time increases. However, two different regimes can be observed, that is $n \sim t$ and $w \sim t^{1/2}$, for $0 \leqslant p < \frac{1}{2}$; while just at p_{1c} one has $n \sim w \sim t^{1/2}$. (ii) The steady-state phase ($\frac{1}{2} < p < p_{2c}$), in which n and w reach finite non trivial ($n > 0$ and $w > 0$) values, but both quantities diverge for $p \to 1/2^+$ as $(p - \frac{1}{2})^{-1}$. (iii) The inactive (or vacuum) state ($p_{2c} \leqslant p \leqslant 1$), for which $n = 0$. At p_{2c} the system exhibits an irreversible phase transition which belongs to the universality class of directed percolation model, so for $p \to p_{2c}^-$, $n \sim (p_{2c} - p)^{\beta_2}$ and $w \sim (p_{2c} - p)^{\beta_2/2}$, with $\beta_2 \simeq 0.277$. Transitions between phases are continuous, however, the transition at p_{1c} (p_{2c}) is reversible (irreversible), respectively.

1. Introduction

Far from equilibrium reaction systems undergoing irreversible phase transitions, generically classified as 'interacting particle systems' [1, 2], is a field that continues to attract great interest from physical and biological scientists. The occurrence of transitions between an active stationary state and an inactive absorbing one is the common feature of many systems which arise in diverse areas such as catalysis (absorbing ≡ poisoned) [3, 4], stochastic growth such as directed percolation (absorbing ≡ no-percolating) [5], dynamic evolution of living societies (absorbing ≡ dead) [6], forest fire propagation (absorbing ≡ fire extinction) [7], damage spreading (absorbing ≡ frozen) [8], etc. Within this context, the contact process, earlier proposed by Harris [9] as a model for an epidemic, is the archetype model for the study of irreversible phase transitions [1, 2].

In the *standard* one-dimensional contact process [1, 9] each site of lattice can be either empty or occupied by only one particle (a monolayer) and the evolution of the system is governed by the following rules. Particles are annihilated at rate p independent of the state of other sites, and empty sites become occupied at rate $\frac{1}{2}$ or 1, provided that one or two nearest-neighbours are occupied, respectively. As there is no spontaneous creation of particles, for large p the system can become trapped in a state with zero particles (the

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vacuum or absorbing state). For sufficiently small p the system reaches, at large times, a steady-state regime with a non-zero average particle density (the active state). At a critical value p_c the system presents a continuous irreversible phase transition which belongs to the same universality class of directed percolation model.

On the other hand, the study of interfaces is one of the most active areas in physics and physical-chemistry (for a review see e.g. [10]). In particular, the structure of a non-equilibrium interface and the dynamic of its roughening are issues of considerable interest. The phenomena of wetting [11], the surface structure of diffusion-limited-aggregates [12], the structure of domain walls in lattice spin models [13], etc, are some examples. Furthermore, the study of rough interfaces upon thin film growth [14] has lead to the formulation of many multilayer growth models involving random deposition of particles, random deposition with surface diffusion, ballistic deposition, etc; see e.g. [15, 16] and for a recent review see [10].

The aim of this work is to propose and study a generalization of the standard contact process by allowing the creation of new particles in each occupied lattice site at rate $(1-p)$. This creation of particles can be thought of as a deposition process in occupied lattice sites. Then, the sites can be occupied by many particles and a multilayer structure appears. The time dependence of both the mean value of particles per lattice site n and its fluctuations w (which characterize the roughness of the interface), are studied for all values of p. It is found that the proposed model exhibits a complex and rich critical behaviour. In fact, it is observed a reversible phase transition between a growing state where the interface diverges and a stationary state where the interface remains finite. These regimes are studied analytically and exact results are derived. Furthermore, an irreversible phase transition between the stationary state and a vacuum absorbing state is also observed. This transition is studied by means of numerical simulations and belongs to the universality class of directed percolation.

2. The model and the Monte Carlo simulation

The multilayer contact process (MCP) is a continuous-time Markov process in which at time t each site i of a lattice (typically the d-dimensional hypercubic lattice, $\mathbb{Z}^d$), is either vacant or occupied by $n_i(t)$ particles.

At each Monte Carlo step one site of the system (for example, site j) is randomly chosen and the following situations may appear: (i) If the chosen site is empty, it is occupied ($n_j = 0 \rightarrow n_j = 1$) with probability m/q, where q is the coordination number of the lattice and m is the number of nearest-neighbour occupied sites. (ii) If the site is occupied, one particle of the column j is either eliminated ($n_j \rightarrow n_j - 1$) with probability p, or one particle is deposited ($n_j \rightarrow n_j + 1$) with probability $1 - p$.

Since particles can only be created by other particles, the vacuum state is absorbing as in the standard contact process. However, in contrast to that process, in the MCP multiple occupancy of lattice sites is allowed, i.e. rule (ii). As it will be shown below, for large times and depending on the value of p, either the number of particles in the system diverge with time, or the system reaches a steady-state regime where the number of particles per lattice site remains bounded.

Exact results for the MCP are derived for the divergency of the average number of particles and the interface width. Close to the vacuum state some exact results are also obtained, but the location of the critical point has to be made by means of a numerical Monte Carlo approach. For this purpose simulations in one-dimensional lattices of size L are performed, assuming periodic boundary conditions. In the algorithm, after each Monte

Carlo step the time t is increased by $1/L$ so that, at each unitary time interval, every site of the lattice is selected, on average, once.

3. An equation for the density of particles

It is possible to derive a differential equation for the global density n as a function of time t, by analysing all possible processes that can take place in a single time step δt.

The global density n is obtained averaging over all lattice sites: $n(t) = \langle n_i(t)\rangle$. Also, let $s_i(t)$ be the occupation number of site i (1 if occupied, 0 otherwise). The mean occupation number of the lattice is then $s(t) = \langle s_i(t)\rangle$.

Knowing $n_i(t)$ it is possible to find an expression for the mean value of the number of particles in site i at time $t + \delta t$ in one dimension,

$$\overline{n_i(t+\delta t)} = n_i\left(1 - \frac{1}{L}\right) + [(n_i + 1)(1-p) + (n_i - 1)p]s_i\frac{1}{L} + (s_{i-1}/2 + s_{i+1}/2)(1 - s_i)\frac{1}{L} \tag{1}$$

where $1/L$ is the probability of choosing site i in a Monte Carlo step and is equal to the time step δt. Notice that the time dependence in n_i and s_i has been dropped for simplicity. equation (1) accounts for the following possibilities: (a) the site i is not selected (first term of the right-hand side), (b) the site i is occupied and the process of deposition or evaporation takes place (second term), and (c) the site i is empty and becomes occupied due to the contact process of occupied neighbouring sites.

Averaging equation (1) over lattice sites, it follows:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = (1-2p)s + \tfrac{1}{2}A \tag{2}$$

where $A = \langle s_{i-1}(1 - s_i) + (1 - s_i)s_{i+1}\rangle$ is the density of pairs of sites that are occupied-empty or empty-occupied. The first term on the right-hand side of equation (2) represents the changes in n due to evaporation of particles with probability p (rate of change equal to $-ps$) and deposition of particles in occupied sites with probability $1 - p$ (rate of change $(1 - p)s$). This term is positive or negative whether p is smaller or greater than $\frac{1}{2}$. The second term is zero or positive and represents the lateral growth in empty sites due to occupied neighbours. Equation (2) can be easily extended to d dimensions changing the definition of A: $A/2 = \langle(1-s_i)\sum_{i'} s_{i'}\rangle/q$, where $\sum_{i'} s_{i'}$ is the sum over nearest neighbours of site i, and q is the coordination number. With this change, all the analytical results presented in sections 4–6 hold.

4. Case $p < p_{1c} = \frac{1}{2}$

For $p < \frac{1}{2}$, from equation (2) it follows that $\mathrm{d}n/\mathrm{d}t > 0$ for all time. The density n always grows and after a certain time there are no more empty sites. For large enough times, $s = 1$, $A = 0$, and $\mathrm{d}n/\mathrm{d}t = 1 - 2p$, so,

$$n = \text{constant} + (1-2p)t \qquad \text{for large } t. \tag{3}$$

The system is reduced to a random evaporation–deposition process with no lateral growth. In figure 1 the prediction of equation (3) is confirmed via numerical simulations. For $p = 0$ the random deposition model [15] is recovered. Since for $p < \frac{1}{2}$ and large times, $A = 0$, there are no correlations between the columns as in the case of a random deposition, and then, the fluctuations $w = \sqrt{\langle n_i^2\rangle - \langle n_i\rangle^2}$ grow as $t^{1/2}$.

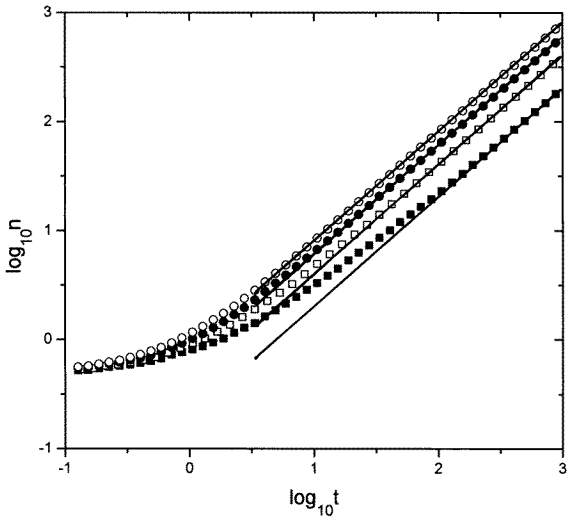

Figure 1. Density n against t in log–log scales for $p < \frac{1}{2}$. The points correspond to numerical simulations and the lines correspond to equation (3) with constant = 0. From top to bottom: $p = 0.1$ (○), $p = 0.2$ (●), $p = 0.3$ (□), and $p = 0.4$ (■). The numerical results of this figure and those shown in figure 3 were obtained using lattices of length $L = 10^4$ and starting with each site of the lattice occupied by one particle with probability $\frac{1}{2}$.

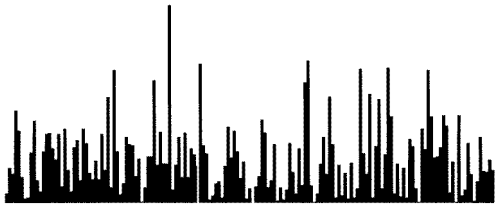

Figure 2. Diagram of the number of particles n_i in a portion of the lattice in the case $p = \frac{1}{2}$. When a fluctuation reaches the bottom of the diagram, a particle can enter the system due to the lateral growth process.

5. Case $p = p_{1c} = \frac{1}{2}$

In this case, from equation (2) it follows,

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{1}{2}A \qquad \text{for } p = p_{1c} = \tfrac{1}{2} \tag{4}$$

this equation indicates that n will always grow, or will be a constant if A were 0. Let us suppose that the lattice is completely occupied (with a given density $n_i = n > 1\ \forall i$), so that $A = 0$. Due to the evaporation with probability $p = \frac{1}{2}$ and the deposition with probability $1 - p = \frac{1}{2}$, every site behaves as a random walk, up or down with equal probabilities. In this process, fluctuations w grow as $t^{1/2}$. There will be a time for which a fluctuation reaches the bottom (see figure 2), and an empty site is created. The empty site is surrounded by occupied sites, and is occupied in average with probability 1 in the unit interval of time because of the lateral grow process. So, when a fluctuation produces an empty site, one has $A > 0$, and n grows (see equation (4)). Of course, fluctuations of the order of n are necessary in order to produce empty sites. The density n (the mean value of the columns in

figure 2) grows if the fluctuations also grow. Since the fluctuations grow as $t^{1/2}$, it follows that $n \sim t^{1/2}$. On the other hand, one can obtain this behaviour working in the continuous limit. The fact that when the fluctuation reaches the bottom the empty site is occupied, corresponds to a random walk with reflecting boundary conditions at the bottom. The probability density of having a column of height y at time t is $P(y,t)$ and the corresponding equation is $\partial P/\partial t = D\ \partial^2 P/\partial y^2$, with the diffusion constant $D = \frac{1}{2}$ in the present case. Then, imposing that $P(y,0) = \delta(y)$, it follows that $P(y,t) = \exp(-y^2/2t)/(\pi t/2)^{1/2}$. Assuming that the space average is equal to the average in configurations, one obtains that $n = \langle n_i \rangle = \int_0^\infty y P(y,t)\,\mathrm{d}y$. Then,

$$n = \sqrt{\frac{2}{\pi}t}. \tag{5}$$

In the same way, one calculates $\langle n_i^2 \rangle$ to find the fluctuation

$$w = \sqrt{\left(1 - \frac{2}{\pi}\right)t}. \tag{6}$$

For other initial conditions, equations (5) and (6) hold asymptotically for large times. From equations (4) and (5),

$$A = \sqrt{\frac{2}{\pi t}}. \tag{7}$$

In the discrete case of our model, the previous results hold also for large times. Figure 3 shows plots of n, w, and A as functions of time in log–log scales. The numerical simulations confirm the mentioned results.

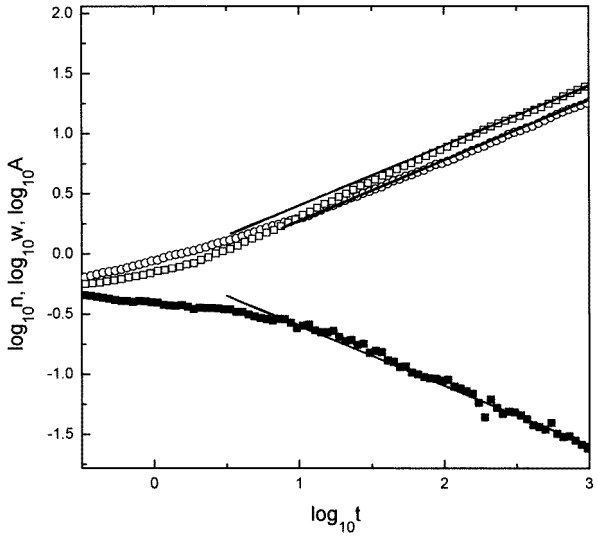

Figure 3. Plot of numerical results of n (□), w (○) and A (■) as functions of time t in log–log scales for $p = \frac{1}{2}$. The upper, middle, and lower straight lines correspond to equations (5)–(7), respectively.

6. Case $p > p_{1c} = \frac{1}{2}$

In this case the evaporation rate is greater than the deposition rate, so the first term in equation (2) is less than or equal to zero. The second term, representing the lateral growth, is zero or positive. A steady state is reached when both processes are balanced, so $\mathrm{d}n/\mathrm{d}t = 0$. For $p > \frac{1}{2}$ and p lower than a critical probability p_{2c}, the density n is greater than zero in the steady state. For $p > p_{2c}$, n, s and A are equal to zero.

It is possible to find relations between n, s and A in the steady state. From equation (2) it follows,

$$s = \frac{A}{4(p - \frac{1}{2})}. \tag{8}$$

Let u_j be the density of sites with j particles. Then, a set of equations for u_j can be derived following the procedure used with equation (1),

$$\begin{aligned}
\frac{\mathrm{d}u_0}{\mathrm{d}t} &= pu_1 - \frac{A}{2} \\
\frac{\mathrm{d}u_1}{\mathrm{d}t} &= -u_1 + pu_2 + \frac{A}{2} \\
\frac{\mathrm{d}u_2}{\mathrm{d}t} &= (1-p)u_1 - u_2 + pu_3 \\
&\vdots \\
\frac{\mathrm{d}u_j}{\mathrm{d}t} &= (1-p)u_{j-1} - u_j + pu_{j+1}.
\end{aligned} \tag{9}$$

In the steady state the time derivatives are equal to zero in equation (9); so a system of linear equations is obtained. It can be proved by induction that the solution of the system is

$$u_j = \frac{A}{2p}\left(\frac{1}{p} - 1\right)^{j-1} \qquad \text{for } j \geqslant 1. \tag{10}$$

The values of u_j are probabilities. Assuming that the space average is equal to the average in configurations, one has $n = \langle n_i \rangle = \sum_{j=1}^{\infty} j u_j$, so

$$n = \frac{sp}{2(p - \frac{1}{2})} \tag{11}$$

where equation (8) has been used to relate A and s. From the probabilities u_j it is possible to calculate the second-order momentum: $\langle n_i^2 \rangle = \sum_{j=1}^{\infty} j^2 u_j$. Now, the mean value of the fluctuations, $w^2 = \langle n_i^2 \rangle - \langle n_i \rangle^2$, is obtained giving

$$w = \frac{\sqrt{sp(1-sp)}}{2(p - \frac{1}{2})}. \tag{12}$$

For $p \to p_{1c}^+$ there are very few empty sites. So, as it can be observed in figure 4, one has $s \to 1$ for $p \to p_{1c}^+$. Then, in this limit one has, from equations (11) and (12), that

$$n \simeq \frac{1}{4(p - p_{1c})} \qquad w \simeq \frac{1}{4(p - p_{1c})} \qquad \text{for } p \to p_{1c}^+, \ (p_{1c} = \tfrac{1}{2}). \tag{13}$$

In figure 5 the behaviour of equation (13) is compared with numerical simulations and the agreement observed is excellent.

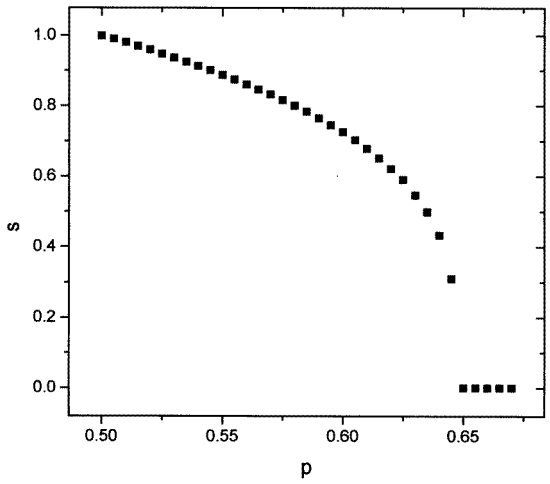

Figure 4. The mean occupation number s against the probability p in the steady state regime. The numerical results of this figure and those shown in figure 5 were obtained at time $t = 5\times10^4$ and using lattices of length $L = 10^4$.

So, for $p \leqslant p_{1c}$ the MCP exhibits a growing phase where both the average global density and the interface roughness diverges with time, however for $p > p_{1c}$ those quantities remain bounded. Notice that this transition is *reversible* in contrast to the transition into the absorbing state which is *irreversible*.

For $p \to p_{2c}^-$, i.e. close to the irreversible transition into the vacuum state, n can be taken as the order parameter, so one expects that $n \sim (p_{2c} - p)^{\beta_2}$, where β_2 is the order parameter critical exponent. From equations (11) and (12),

$$s \sim (p_{2c} - p)^{\beta_2} \qquad w \sim (p_{2c} - p)^{\beta_2/2} \qquad \text{for } p \to p_{2c}^-. \tag{14}$$

For $p > p_{2c}$, as stated before, n, s, A and w are equal to zero.

7. Analysis of the irreversible transition to the vacuum state

As discussed above, just at p_{2c} the system exhibits a continuous irreversible phase transition from the active stationary state to an inactive vacuum state. The precise location of the critical point as well as the evaluation of relevant critical exponents cannot be achieved using the analytic solution developed in the previous sections. This task can be done by mean of computer simulations. However, due to fluctuations of the stochastic system close to criticality, standard approaches are not useful because the system can irreversibly be trapped by the vacuum state. This shortcoming can be avoided performing dynamic epidemic analysis introduced by Grassberger and de la Torre [17]. For this purpose, one generates a lattice completely empty except for a nearest-neighbour pair of occupied sites at the centre of the lattice, i.e. a configuration very close to the vacuum state. Starting from this configuration a large number of independent runs are performed for different values of p close to criticality. The measured quantities are: (i) the survival probability $P(t)$, that is, the probability that the lattice has not entered in the vacuum state after time t; (ii) the average number of occupied sites $N(t)$; and (iii) the average mean square distance, $R^2(t)$, over which the occupied sites had spread. Results are not affected by finite size artefacts since lattices are taken large enough such as the epidemic never reaches the edges. At the

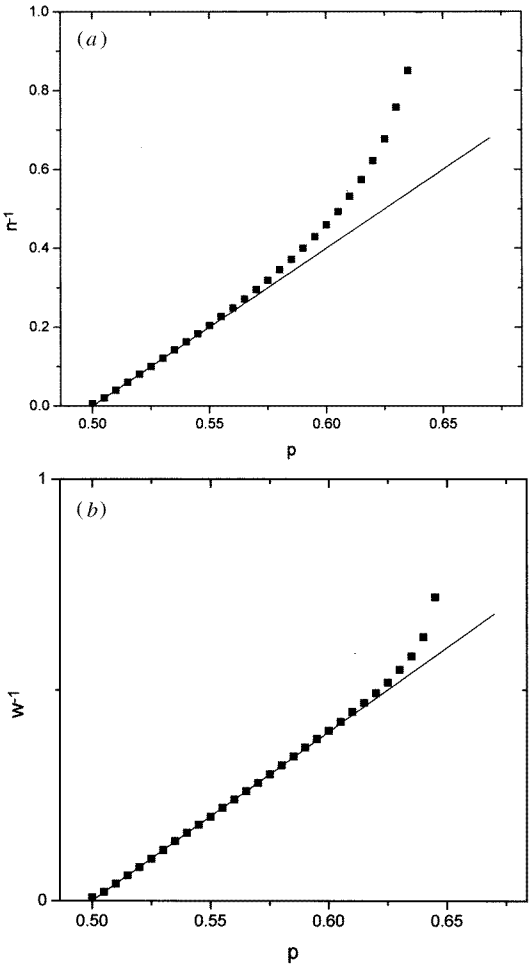

Figure 5. Numerical results of (*a*) n^{-1} and (*b*) w^{-1} against the probability p in the steady state regime. In a region near $p = \frac{1}{2}$ the behaviour is well represented by the lines which correspond to equation (13).

critical point and for large values of t, the following scaling ansatz should hold:

$$P(t) \propto t^{-\delta} \tag{15}$$

$$N(t) \propto t^{\eta} \tag{16}$$

$$R^2(t) \propto t^{z}. \tag{17}$$

Thus, when $p = p_{2c}$, log–log plots of $P(t)$, $N(t)$ and $R^2(t)$ will asymptotically show a straight line behaviour, while off-critical points will exhibit curvature (see figure 6). With the aid of these plots the critical point is located at $p_{2c} = 0.6473 \pm 0.0003$ and from the slopes of the plots the estimates for the critical exponents are

$$\delta = 0.1605 \pm 0.0005 \qquad \eta = 0.3083 \pm 0.0005 \qquad z = 1.264 \pm 0.005.$$

These figures are in excellent agreement with the values obtained by mean of computer simulations for the standard contact process, i.e.

$$\delta = 0.161 \pm 0.003 \qquad \eta = 0.305 \pm 0.005 \qquad z = 1.257 \pm 0.005$$

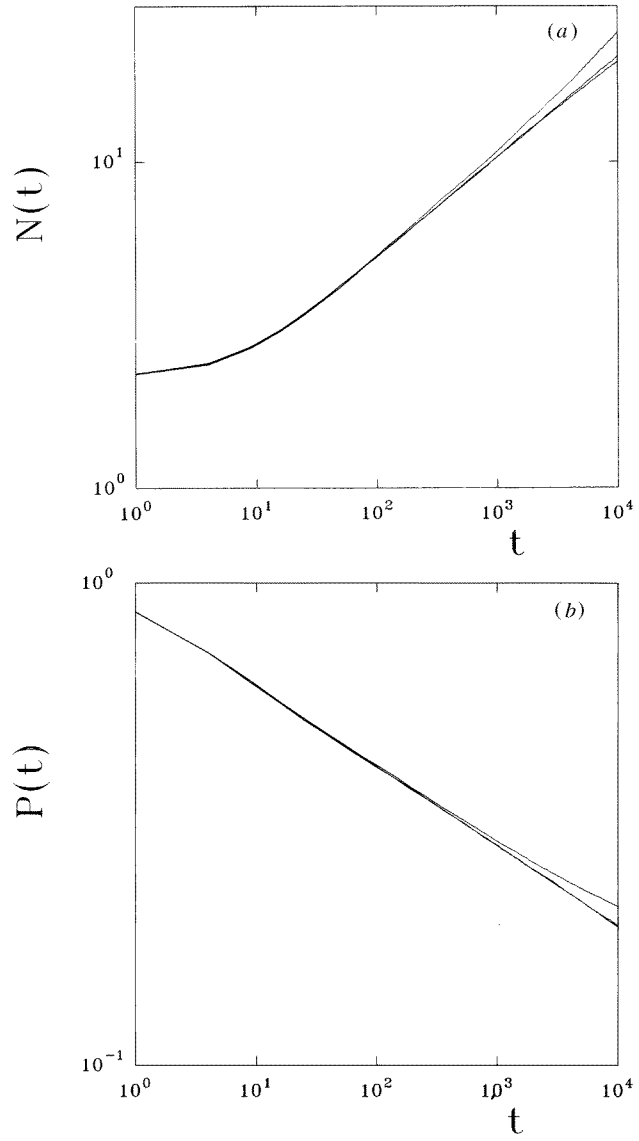

Figure 6. Log–log plots of (a) the number of occuppied sites $N(t)$; (b) the survival probability $P(t)$; and (c) the average square distance of spreading $R^2(t)$ versus time t, obtained close to criticality. Upper curves: $p = 0.6470$ (supercritical), medium curves: $p = 0.6473$ (critical) and lower curves $p = 0.6474$ (subcritical). Averages are taken over 10^5 different initial configurations.

as well as with the exponents corresponding to directed percolation in $(1 + 1)$ dimensions,

$$\delta = 0.160 \qquad \eta = 0.308 \qquad z = 1.265.$$

It should be noted that the dynamic exponents are not fully independent since the scaling relation $\Delta = dz - 2\eta - 4\delta = 0$ is expected to hold [17]. Using the obtained exponents one obtains $\Delta = 0.005$, which in fact shows the validity of the scaling relationship.

After determining accurately the critical point, one can gain further insight of the critical

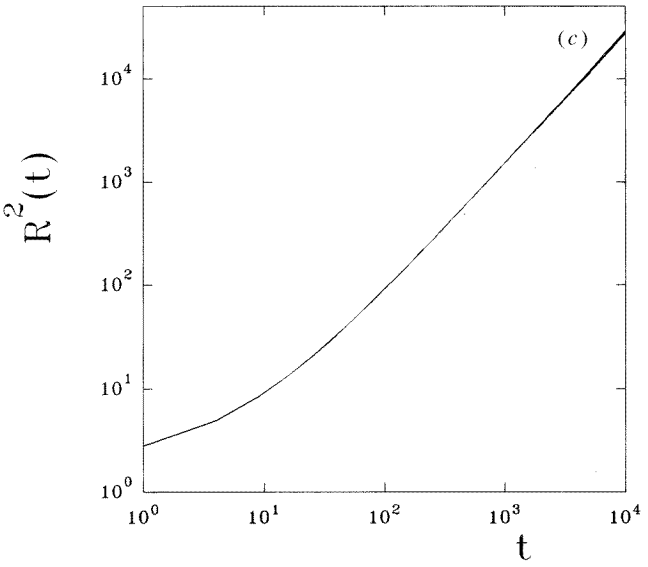

Figure 6. (Continued)

behaviour of the model performing epidemic analysis within the subcritical (vacuum) state where the following scaling law should hold [17],

$$N(t) \sim t^{\eta}\Psi(|p - p_c|t^{1/\nu_{\|}}) \tag{18}$$

where $\nu_{\|}$ is the correlation length exponent in the so-called time-direction. In the vacuum state the correlations are short-ranged and one therefore expects $N(t)$ to decay exponentially. This can only happen if for $\Delta p = p - p_c \to 0$ and $t \to \infty$, the scaling function Ψ behaves as

$$\Psi(y) \propto y^{-\eta\nu_{\|}} \exp(-ky^{\nu_{\|}}) \tag{19}$$

where k is a constant. Therefore, using equations (18) and (19) it follows

$$N(t) \sim (\Delta p)^{-\eta\nu_{\|}} \exp[-k(\Delta p)^{\nu_{\|}}t]. \tag{20}$$

From equation (20) it follows that in the vacuum state $N(t)$ should decay exponentially and that the decay constant λ, governing the long-time behaviour is proportional to $(\Delta p)^{\nu_{\|}}$. Figure 7(*a*) shows that in plots of $\ln N(t)$ versus t one can see asymptotically a straight line behaviour with slope λ. In fact, this statement is confirmed in figure 7(*b*) where a log–log plot of λ versus Δp gives a straight line and from the slope one can evaluate the exponent $\nu_{\|} \cong 1.728 \pm 0.0052$, in agreement with the accepted value for directed percolation in $(1+1)$ dimensions, i.e. $\nu_{\|} \cong 1.733$ [18].

Furthermore, using the scaling relationship [17] $\beta = \nu_{\|}\delta$, one can obtain an estimation of the order parameter critical exponent which gives $\beta_2 = 0.2773$ in agreement with the best available estimated for directed percolation obtained by series analysis, i.e. $\beta = 0.2763(6)$ (where the error bars account for β values determined using different lattices and for bond and site directed percolation) [19]. So, according to equation (14), the fluctuation of the interface vanishes with exponent $\beta_2/2$.

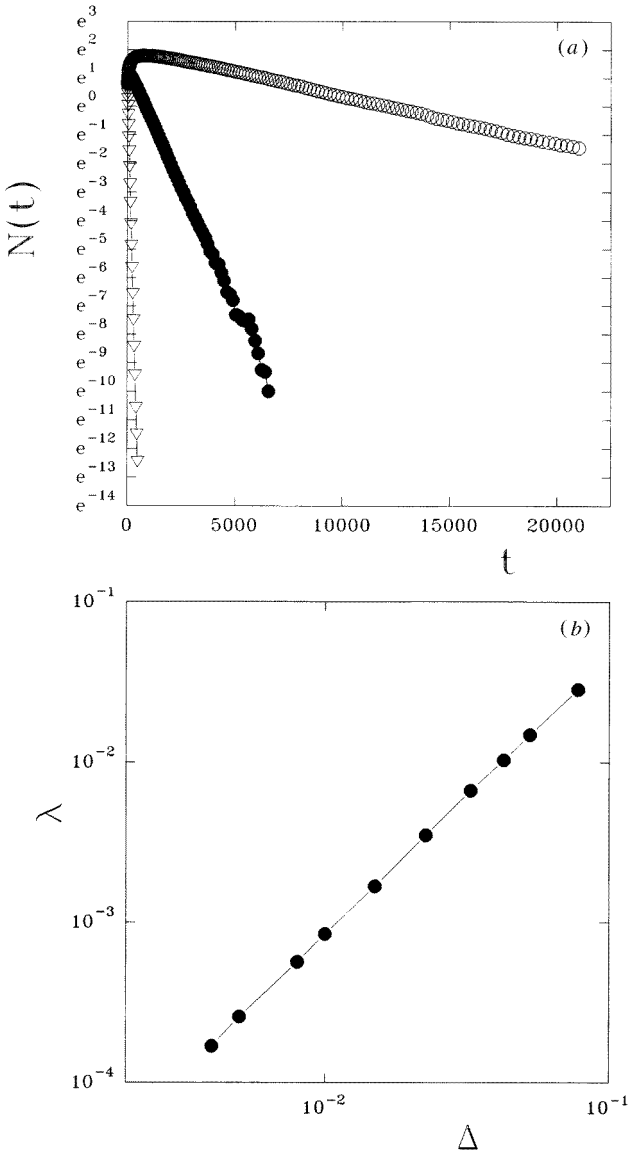

Figure 7. (*a*) *ln*-lineal plots of the number of occupied sites $N(t)$ versus time t, obtained within the subcritical regime for different values of p. Upper curve $p = 0.6513$, medium curve $p = 0.6623$ and lower curve $p = 0.7250$. The critical probability is $p_{2c} = 0.6473$. Averages are taken over 10^6 different initial configurations. (*b*) Log–log plots of λ versus Δp. The straight line has slope $\nu_{\parallel} = 1.728$.

8. Conclusions

A multilayer contact model defined in section 2 is introduced and studied by means of analytical and numerical approaches in one dimension. The evolution of the system as a function of time t depends on a free parameter p, which governs the creation (deposition) and annihilation (evaporation) of a single particle (at rate p and $(1 - p)$, respectively) in

each occupied site. Due to the deposition mechanism on already occupied sites multilayer structures can appear, in contrast to standard contact process.

By means of analytical results and Monte Carlo simulations it is found that in the asymptotic time regime the model exhibits a rich critical behaviour with three different phases:

(i) The growing phase for $0 \leqslant p \leqslant p_{1c} = \frac{1}{2}$. Here the mean value of particles per lattice site n and its fluctuations w diverge with time according to $n \sim (1-2p)t$ and $w \sim t^{1/2}$; for $0 \leqslant p < \frac{1}{2}$. This behaviour corresponds to a random deposition model with an effective rate of deposition $(1-2p)$. However, just at p_{1c}, $n \sim w \sim t^{1/2}$. These results, obtained analytically have been confirmed by Monte Carlo simulations.

(ii) The steady-state phase ($p_{1c} < p < p_{2c}$), in which n and w reach finite non-zero values. For $p \to p_{1c}^{+}$, n and w diverge as $(p-\frac{1}{2})^{-1}$; while for $p \to p_{2c}^{-}$, n and w vanish as $\sim (p_{2c}-p)^{\beta_2}$ and $\sim (p_{2c}-p)^{\beta_2/2}$, respectively, where β_2 is the order parameter critical exponent of the universality class of directed percolation.

(iii) The inactive (or vacuum) state ($p_{2c} < p \leqslant 1$), for which $n = w = 0$.

The transition between the growing phase and the stationary one is continuous and *reversible*. In contrast, the transition between the stationary phase and the absorbing state is *irreversible*. It is also shown, by means of an epidemic spreading analysis that the continuous irreversible phase transition at p_{2c} belongs to the same universality class as directed percolation.

As a final remark, let us comment that the present model depends on a single parameter p, which corresponds to the evaporation rate of the standard contact process. The deposition probability is $1-p$. One can generalize the present model introducing an additional independent paramenter, the deposition probability q. Then, the probability of no change in the number of particles is $1-p-q$. In the (p,q) plane, this new model is defined within the triangle given by $p \geqslant 0$, $q \geqslant 0$, and $p+q \leqslant 1$. The lines $q=0$ and $q=1-p$ in this triangle correspond to the standard contact process and to the model defined in section 2, respectively. In this generalization, equation (2) changes to $\mathrm{d}n/\mathrm{d}t = (q-p)s + A/2$ (for a one dimensional system). Then, for $q > p$ one obtains $n = \text{constant} + (q-p)t$, for large t ($n \sim t$ and $w \sim t^{1/2}$, see equation (3)); and $p_{1c}(q) = q > 0$ is a critical line between the growth phase and the steady-state regime ($n \sim w \sim t^{1/2}$, see equation (4)). In the region $p > q$ a critical line $p_{2c}(q)$ appears, which corresponds to an irreversible transition between stationary and vacuum states. This transition belongs to the universality class of directed percolation.

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